Exercise 37

Find y'' by implicit differentiation.

$$\sin y + \cos x = 1$$

Solution

Differentiate both sides with respect to x.

$$\frac{d}{dx}(\sin y + \cos x) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(\sin y) + \frac{d}{dx}(\cos x) = 0$$

$$(\cos y) \cdot \frac{d}{dx}(y) + (-\sin x) = 0$$

$$y' \cos y - \sin x = 0$$

Solve for y'.

$$y' = \frac{\sin x}{\cos y}$$

Differentiate both sides with respect to x to get y''.

$$\begin{aligned}
\frac{d}{dx}(y') &= \frac{d}{dx}\left(\frac{\sin x}{\cos y}\right) \\
y'' &= \frac{\left[\frac{d}{dx}(\sin x)\right]\cos y - \left[\frac{d}{dx}(\cos y)\right]\sin x}{(\cos y)^2} \\
&= \frac{(\cos x)\cos y - \left[(-\sin y) \cdot \frac{d}{dx}(y)\right]\sin x}{\cos^2 y} \\
&= \frac{\cos x \cos y + y' \sin x \sin y}{\cos^2 y} \\
&= \frac{\cos x \cos y + \left(\frac{\sin x}{\cos y}\right)\sin x \sin y}{\cos^2 y} \cdot \frac{\cos y}{\cos y} \\
&= \frac{\cos x \cos^2 y + \sin^2 x \sin y}{\cos^3 y}
\end{aligned}$$